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Non Homo. Recc. Relation with const. coeffs

$$f(n)+c_1 f(n-1)+c_2 f(n-2)+\dots+c_k f(n-k)=q(n)$$

Case III :→ $q(n)$ is a polyn of degree m.

let $q(n)=q_0+q_1 n+\dots+q_m n^m$

For the particular soln, take

$$f(n)=d_0+d_1 n+\dots+d_m n^m$$

But if particular solution has terms similar to homo soln then multiply the particular soln by n.

Q:→ Solve the recurrence relation

$$S(n)+5S(n-1)+6S(n-2)=3n^2-2n+1 \quad \text{——} (1)$$

Sol:→ Homo Soln $(S^h(n))$

Associated Homo eqn $S(n)+5S(n-1)+6S(n-2)=0$ — (2)

For char eqn, take $S(n)=a^n$ in (2)

$$a^n+5a^{n-1}+6a^{n-2}=0$$

$$\Rightarrow a^2+5a+6=0$$

$$\Rightarrow (a+2)(a+3)=0$$

$$\Rightarrow a=-2,-3$$

$$\therefore S^h(n)=A(-2)^n+B(-3)^n \checkmark$$

Particular Soln $(S^p(n))$

As $q(n)=3n^2-2n+1$ which is a polyn of degree 2

For particular soln, take $S(n) = d_0 + d_1 n + d_2 n^2$ in (1)

$$S(n) + 5S(n-1) + 6S(n-2) = 3n^2 - 2n + 1$$

$$\Rightarrow [d_0 + d_1 n + d_2 n^2] + 5[d_0 + d_1(n-1) + d_2(n-1)^2]$$
$$+ 6[d_0 + d_1(n-2) + d_2(n-2)^2] = 3n^2 - 2n + 1$$

$$\Rightarrow [1+5+6]d_0 + [n + 5(n-1) + 6(n-2)]d_1$$
$$+ [n^2 + 5(n-1)^2 + 6(n-2)^2]d_2 = 3n^2 - 2n + 1$$

$$\Rightarrow 12d_0 + (12n - 17)d_1 + (12n^2 - 34n + 29)d_2 = 3n^2 - 2n + 1$$

$$\Rightarrow (12d_0 - 17d_1 + 29d_2) + (12d_1 - 34d_2)n + 12d_2 n^2 = 3n^2 - 2n + 1$$

Equate coeff of n^0, n^1 and n^2

$$12d_0 - 17d_1 + 29d_2 = 1 \Rightarrow d_0 = \frac{71}{288}$$
$$12d_1 - 34d_2 = -2 \Rightarrow d_1 = \frac{13}{24}$$
$$12d_2 = 3 \Rightarrow d_2 = \frac{1}{4}$$

$$\therefore S^p(n) = d_0 + d_1 n + d_2 n^2$$
$$= \frac{71}{288} + \frac{13}{24}n + \frac{1}{4}n^2$$

Complete soln

$$S(n) = S^h(n) + S^p(n)$$
$$= A(-2)^n + B(-3)^n + \frac{71}{288} + \frac{13}{24}n + \frac{1}{4}n^2 \quad \text{Ans}$$

Q:→ Solve the recurrence relation

$$S(k) - 4S(k-1) + 3S(k-2) = k^2 \quad \text{———— (1)}$$

Sol:→ Homo soln $(S^h(k))$

Associated Homo eqn $S(k) - 4S(k-1) + 3S(k-2) = 0$ — ②

For char eqn, take $S(k) = a^k$ in ②

$$a^k - 4a^{k-1} + 3a^{k-2} = 0$$

$$\Rightarrow a^2 - 4a + 3 = 0$$

$$\Rightarrow (a-1)(a-3) = 0$$

$$\Rightarrow a = 1, 3$$

$$\therefore S^h(k) = A(1)^k + B(3)^k$$

$$= \underline{A} + B(3)^k$$

Particular soln $(S^p(k))$

$g(k) = k^2$ which is a polyn of degree 2.

For particular soln, we take

$S(k) = \underline{d_0} + d_1 k + d_2 k^2$

the term d_0 in particular solution is similar to constant term in homo soln. Then we're to multiply by k.

$$\therefore S(k) = d_0 k + d_1 k^2 + d_2 k^3$$

From the given recc. relation and $S(k) = d_0 k + d_1 k^2 + d_2 k^3$

$$S(k) - 4S(k-1) + 3S(k-2) = k^2$$

$$\Rightarrow [d_0 k + d_1 k^2 + d_2 k^3] - 4[d_0(k-1) + d_1(k-1)^2 + d_2(k-1)^3] + 3[d_0(k-2) + d_1(k-2)^2 + d_2(k-2)^3] = k^2$$

$$\Rightarrow [k - 4(k-1) + 3(k-2)]d_0 + [k^2 - 4(k-1)^2 + 3(k-2)^2]d_1 + [k^3 - 4(k-1)^3 + 3(k-2)^3]d_2 = k^2$$

$$\Rightarrow [k - 4k + 4 + 3k - 6]d_0 + [k^2 - 4k^2 + 4 + 8k + 3k^2 - 12k + 12]d_1$$

$$\Rightarrow \quad + [k^3 - 4k^3 + 12k^2 - 12k + 4 + 3k^3 - 18k^2 + 36k - 24] d_2 = k^2$$

$$\Rightarrow \quad -2d_0 + (-4k+8)\, d_1 + (-6k^2 + 24k - 20)\, d_2 = k^2$$

$$\Rightarrow \quad (-2d_0 + 8d_1 - 20d_2) + (-4d_1 + 24d_2)\, k - 6d_2 = k^2$$

equate coeff of k^0, k^1 and k^2

$$-2d_0 + 8d_1 - 20d_2 = 0 \quad \Rightarrow \quad d_0 = -7/3$$

$$-4d_1 + 24d_2 = 0 \quad \Rightarrow \quad d_1 = -1$$

$$-6d_2 = 1 \quad \Rightarrow \quad d_2 = -\frac{1}{6}$$

$$\therefore \quad S^p(k) = d_0 + d_1 k + d_2 k^2$$

$$= -\frac{7}{3} - k - \frac{1}{6} k^2$$

Complete soln

$$S(k) = S^h(k) + S^p(k)$$

$$= A + 3^k - \frac{7}{3} - k - \frac{k^2}{6} \quad \text{Ans}$$

Excercise:

1) Solve

$$y(n+2) + 2y(n+1) - 15y(n) = 6n + 10$$

$$y(0) = 1, \quad y(1) = -\frac{1}{2}$$

2) Solve

$$y(n) - 3y(n-1) + 2y(n) = n^2$$

$$y(0) = 0, \quad y(1) = 0$$